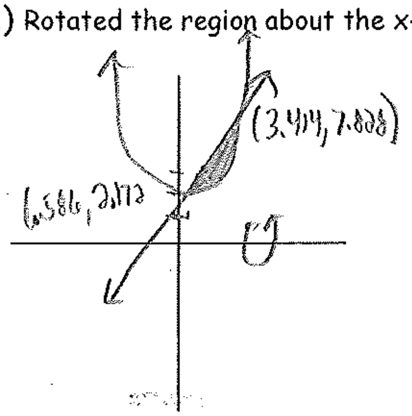
Calculus 1 Volume of Solids of Rotation Name____________________

Worksheet ~ Method of Shells (B) Date____________________

#1-4: Calculate the volume using the requested method for the region bounded by: $y = \frac{1}{2}x^2 + 2$ *and* $y = 2x + 1$

1) Rotated the region about the x-axis and use the method of washers.



Area: TB
Rotate: TB

$$\int_{.586}^{3.414} \pi\left((2x+1)^2 - (\tfrac{1}{2}x^2+2)^2\right)dx$$

$$4x^2+4x+1 - \left(\tfrac{1}{4}x^4 + 2x^2 + 4\right)$$

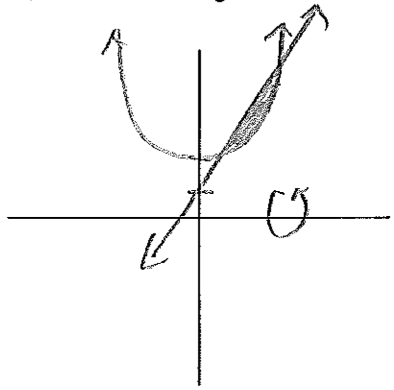
$$\int_{.586}^{3.414} \pi\left(-\tfrac{1}{4}x^4 + 2x^2 + 4x - 3\right)dx$$

$$\pi\left(-\tfrac{1}{20}x^5 + \tfrac{2}{3}x^3 + 2x^2 - 3x\right)\Big|_{.586}^{3.414}$$

$$\pi(17.348) = \boxed{54.499}$$

Calc

2) Rotate the region about the x-axis and use the method of shells.



Area: RL
Rotate: TB

$\int 2\pi y(R-L)$

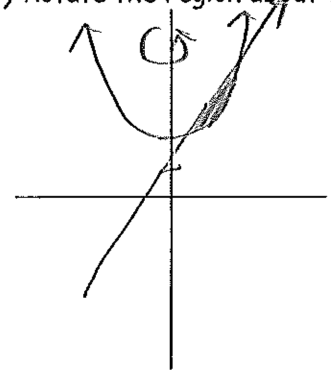
$y = 2x+1$, $y-1 = 2x$, $.5y - .5 = x$ | $y = \frac{1}{2}x^2+2$, $y-2 = \frac{1}{2}x^2$, $2y-4 = x^2$, $\sqrt{2y-4} = x$

$$\int_{2.172}^{7.828} 2\pi y\left(\sqrt{2y-4} - (.5y-.5)\right)dy$$

$$\boxed{54.499}$$

Calc

3) Rotate the region about the y-axis and use the method of washers



Area: RL
Rotate: RL

$(\frac{1}{2}y - \frac{1}{2})(\frac{1}{2}y - \frac{1}{2})$

$$\boxed{23.695}$$

$$\int_{2.172}^{7.828} \pi\left((\sqrt{2y-4})^2 - (.5y-.5)^2\right)dy$$

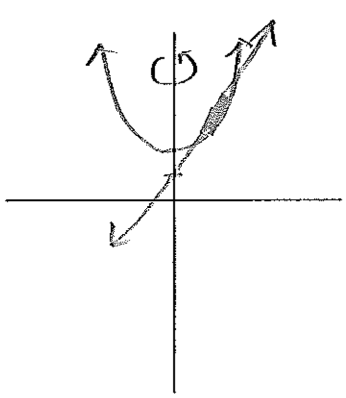
$$2y-4 - \left(\tfrac{1}{4}y^2 - \tfrac{1}{2}y + \tfrac{1}{4}\right)$$

$$\int_{2.172}^{7.828} \pi\left(-\tfrac{1}{4}y^2 + 2.5y - 4.25\right)dy = \pi\left(-\tfrac{1}{12}y^3 + \tfrac{5}{4}y^2 - 4.25y\right)\Big|_{2.172}^{7.828}$$

5/2

Calc

4) Rotate the region about the y-axis and use the method of shells



Area: TB
Rotate: RL

$\int 2\pi x(T-B)$

$$\int_{.586}^{3.414} 2\pi x\left((2x+1) - (\tfrac{1}{2}x^2+2)\right)dx$$

$$\int_{.586}^{3.414} 2\pi x\left(2x - 1 - \tfrac{1}{2}x^2\right)dx$$

$$\int_{.586}^{3.414} 2\pi\left(2x^2 - x - \tfrac{1}{2}x^3\right)dx$$

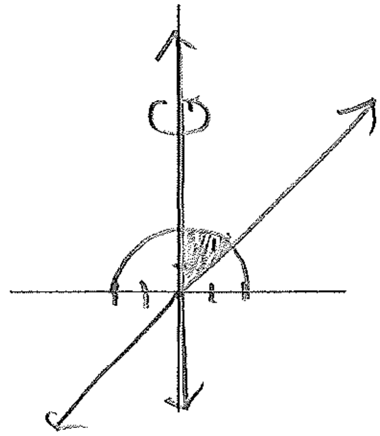
$$2\pi\left(\tfrac{2}{3}x^3 - \tfrac{1}{2}x^2 - \tfrac{1}{8}x^4\right)\Big|_{.586}^{3.414}$$

$$2\pi(3.771)$$

$$\boxed{23.695}$$

#5-8: Use any appropriate method to solve for the volume of rotation.

Calc 5) $y=\sqrt{4-x^2}$ $\quad y=x$ $\quad$ *and* $\quad x=0$ $\qquad$ about the y-axis



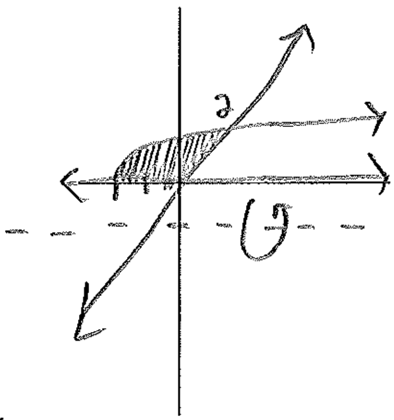
Area: TB ∴ shell $\quad \int 2\pi x(T-B)$

Rotate: RL

$$\int_0^{1.414} 2\pi x\left(\sqrt{4-x^2}-x\right)dx$$

$$\boxed{4.907}$$

Calc 6) $y=\sqrt{x+2}$ $\quad y=x$ $\quad$ *and* $\quad y=0$ $\qquad$ about the line $y=-1$



Area: RL ∴ shell $\quad \int 2\pi y(R-L)$

Rotate: TB

$$\int_0^2 2\pi(y--1)\left(y-(y^2-2)\right)dy$$

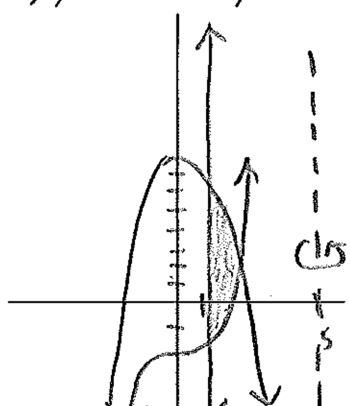
$$\int_0^2 2\pi(y+1)(y-y^2+2)\,dy \qquad \boxed{37.699}$$

$y=\sqrt{x+2}$

$y^2=x+2$

$y^2-2=x$

Calc 7) $y=x^3-2$ $\quad y=-x^2+8$ $\quad$ *and* $\quad x=1$ $\qquad$ about the line $x=5$



Area: TB ∴ shell $\quad \int 2\pi x(T-B)$

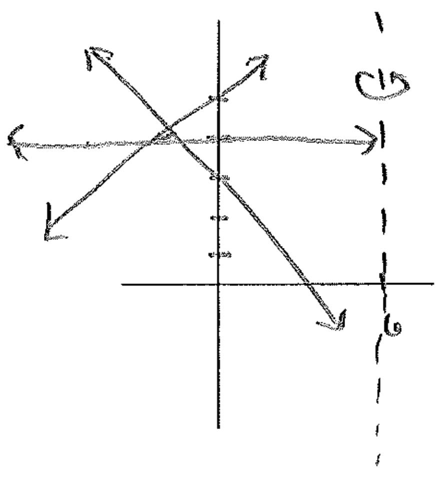
Rotate: RL

$$\int_1^{1.867} 2\pi(5-x)\left(\overbrace{(-x^2+8)}^{T}-\overbrace{(x^3-2)}^{B}\right)dx$$

$$\int_1^{1.867} 2\pi(5-x)\left(-x^2+10-x^3\right)dx$$

$$\boxed{93.8}$$

Calc 8) $y=-2x+3$ $\quad y=x+5$ $\quad$ *and* $\quad y=4$ $\qquad$ about the line $x=6$



Area: RL ∴ washer

Rotate: RL

$$\int_4^{4.333} \pi\left(\left(6-(y-5)\right)^2-\left(6-(-.5y+1.5)\right)^2\right)dy$$

$$\int_4^{4.333} \pi\left((11-y)^2-(4.5+.5y)^2\right)dy$$

$$\boxed{3.52}$$

$y=-2x+3$ | $y=x+5$

$y-3=-2x$ | $y-5=x$

$-.5y+1.5=x$